

(d) Show that the total number of allowed modes in the *first Brillouin zone* of a one dimensional diatomic lattice is equal to $2N$, the total number of degrees of freedom.

3. (a) (i) Mention various factors affecting the intensity of the diffracted X-ray beam from a crystal. (ii) State Bloch theorem.

(b) Show that 5-fold rotational symmetry is not allowed in crystal world.

(c) What is crystal momentum? (2+2+3+1)

4. (a) From the dispersion relation of a one dimension chain of monoatomic lattice on N atoms. Considering only the nearest neighbour interaction show that the density of modes is given by $D(\omega) = \frac{2N}{\pi} \cdot \frac{1}{(\omega_m^2 - \omega^2)^{1/2}}$. where, ω_m is the maximum frequency.

(b) Let the optical phonon branch obeys the relation $\omega(k) = \omega_o - Ak^2$, near $k=0$ in three dimensions. Show that $D(\omega) = \left(\frac{L}{2\pi}\right)^3 \frac{2\pi}{A^{3/2}} (\omega_o - \omega)^{1/2}$ for $\omega < \omega_o$ and $D(\omega) = 0$ for $\omega > \omega_o$.

(c) What is Van Hove's singularity? (4+3+1)

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Internal Assessment-10

2018

M.Sc.

1st Semester Examination

PHYSICS

PAPER – PHS-102 (Gr. – A + B)

Full Marks : 50

Time : 2 Hours

Use separate answer scripts for Group A and Group B

(Quantum Mechanics I – PHS 102A)

Answer Q1, Q2 and any one from Q3 and Q4

1. Answer any two bits:

2x2 = 4

(a) A three-level quantum system has energy eigenvalues 0, 1, 2 MeV. If the probabilities for the system, at time t , to be in the first two eigenstates are 49% and 36% respectively, write down the wave function $\psi(\vec{r}, t)$ for the system.

(b) Prove that the time evolution operator is unitary.

(c) Comment on the collapse of a wave function.

(d) Find the degeneracy of three dimensional isotropic harmonic oscillator in the n -th eigenstate.

2. Answer any two bits:

2x4 = 8

(a) A rigid body with moment of inertia I_z rotates freely in the x - y plane. Let φ be the angle between the x -axis and the rotator axis. Find the energy eigenvalues and the corresponding eigenfunctions.

(b) Define parity operator. Deduce the parity of a hydrogenic wave function ψ_{nlm} .

(Turn Over)

(c) (i) Discuss the condition for the operator \mathbf{A} to be unitary: $(1 + i\mathbf{A}) / (1 - i\mathbf{A})$.

(ii) $\psi(x)$ is a solution of the Schrodinger equation for a free particle of mass m in one dimension where $\psi(x) = A \exp(-x^2/a^2)$. Find the probability amplitude of momentum space, $\psi(p)$. (2+2)

(d) Consider the perturbation $H' = \frac{1}{2}bx^2$ to the one dimensional harmonic oscillator. Calculate the first order energy shift to the ground state and compare it with the exact result. What is the validity of such perturbation to the energy eigen values?

3. (a) Write down the effective potential in radial differential equation for H-atom and explain the significance of different parts. (1+1+1)

(b) Is xp_x hermitian? Explain your reasoning. What is its expectation value in the ground state of harmonic oscillator? Use this operator to prove the virial theorem in one dimension. (2+1+2)

4. (a) An electron is trapped inside an infinite spherical well $V(r) = 0$ for $r < a$ and ∞ for $r > a$. Using the radial Schrodinger equation, determine the bound eigenenergies and the corresponding normalized wave functions for the case $l = 0$. (2+2)

(b) Calculate the expectation value of the operator in the N-representation with respect to the state $|n\rangle$ (i.e. $\langle n|\mathbf{X}^4|n\rangle$). (4)

(Continued)

(Solid State Physics I – PHS 102B)
Answer Q1, Q2 and any one from Q3 and Q4

1. Answer any two bits:

2X2 = 4

(a) Prove that optical branch does not arise in case of monoatomic lattice.

(b) Iron pyrite has fcc structure with $a = 5.42 \text{ \AA}$. If Fe- K_α radiation is used, at what angle do we expect a (300) line? On an actual photograph a line occurs at this angle if radiation is unfiltered. How may this be accounted for? (λ for Fe- $K_\alpha = 1.937 \text{ \AA}$ and Fe- $K_\beta = 1.757 \text{ \AA}$).

(c) The primitive lattice translational vector of hexagonal space lattice may be taken as $\vec{a} = \left(\frac{\sqrt{3}}{2}a\right)\hat{i} + \frac{1}{2}a\hat{j}$, $\vec{b} = -\left(\frac{\sqrt{3}}{2}a\right)\hat{i} + \frac{1}{2}a\hat{j}$, $\vec{c} = c\hat{k}$. Show that the lattice is its own reciprocal, but with a rotation of axis.

(d) What do you mean by *phonons*? Which statistical distribution do they follow?

2. Answer any two bits:

2X4 = 8

(a) Deduce the dispersion relation for a monoatomic linear lattice of N -atoms and show it graphically.

(b) Show that the c/a ratio of an ideal hexagonal closed pack structure is 1.633. 'If c/a is significantly larger than this value, the crystal structure may be thought as composed of planes of closely packed atoms where the planes being loosely stacked' – justify the statement.

(c) The energy wave vector of a 1D crystal is given by $E = E_0 - \alpha - 2\beta \cos ka$.
(i) Find the value of k at which the velocity of electron is maximum. (ii) Find the difference between the top and bottom of the energy band (iii) Obtain the effective mass of the electron at the bottom at top of the band.

(Turn Over)